

Renormalization of Non-locally Regularized BRST-anti-BRST theories

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Abstract

An extension of the non-local regularization scheme is formulated in the $\text{Sp}(2)$ symmetric Lagrangian BRST quantization framework. It generates a systematic treatment of the anomalous quantum master equations and allows to subtract the divergences as well as to calculate genuine higher loop BRST and anti BRST anomalies.

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1 Introduction

The quantization methods based on the BRST symmetry [1]-[5] are acknowledged as powerful techniques in studying gauge theories. In addition, the $\text{Sp}(2)$ extended version of the Lagrangian formalism [6], [10] provided the appropriate treatment of both BRST and anti-BRST symmetries. However, the quantization stage, the anomalies and renormalization problems were less addressed in this second framework until now, although several renormalization procedures (using BPHZ [9] or dimensional regularization [16]) have been completed, but only in the field-antifield case.

In this paper we propose a generalization of the non-local regularization scheme [7],[8],[14] to the $\text{Sp}(2)$ symmetric Lagrangian formulation of the BRST quantization procedure, including a more complete treatment of the renormalization stage and enforcing the $\overline{\Delta}^a$ -operators to act only on non-local expressions at regularized level. The main difficulty of such a regularization technique arises from the degenerate antibracket structure of the formalism and from the non-symmetric roles of the antifields $\phi_{\alpha a}^*, \overline{\phi}_\alpha$ which correspond to every given field ϕ^α .

This problem can be solved by two types of strategies. One of them [13] was previously constructed in detail, based on the degeneration itself, by keeping the entire set of fields and antifields $(\phi^\alpha, \phi_{\alpha a}^*, \overline{\phi}_\alpha)$ as required by the standard method. On the other hand, one can rely on the larger but perfectly symmetric structure of the phase space [11] endowed with Darboux type coordinates where $\phi^\alpha, \pi_{\alpha a}$ (including thus the Nakanishi-Lautrup fields) are considered fields and $\phi_{\alpha a}^*, \overline{\phi}_\alpha$ are treated as the conjugated antifields.

In these coordinates, the antibrackets and the Δ^a, V^a operators are defined as in [11]. The generalization of the proposed regularization scheme involves a set of major differences with respect to the field-antifield case. The anomalous master equation structure loses its well known and useful symmetry exhibited in the BV formalism. The set of generalised Wess-Zumino conditions is more complex and the restrictions imposed to the counter-terms have more involved cohomological consequences.

The paper is organized as follows. In the second section, we give the non-local form of the action which may be used in the perturbation theory calculations for a gauge theory with a known proper solution of the $\text{Sp}(2)$ classical master equations. In the section 3, the general algorithm for treating the genuine BRST anti BRST higher order anomalies is provided, after the

divergences subtraction is completed, at each perturbative stage. The paper concludes with the discussion of certain open problems and consequences.

2 Sp(2) Non-local regularization

Our starting point is a gauge theory having the proper solution $S(\phi, \phi_a^*, \bar{\phi}, \pi_{\alpha a})$ of the classical master equations ($a = 1, 2$) in Sp(2) Lagrangian formalism:

$$\frac{1}{2}(S, S)^a + V^a S = 0 \quad (1)$$

which may be written as the sum of a free and interaction contributions:

$$S(\phi, \phi_a^*, \bar{\phi}, \pi_{\alpha a}) = F(\phi) + I_{cl}(\phi, \phi_a^*, \bar{\phi}, \pi_{\alpha a}) \quad (2)$$

where

$$F(\phi) = \frac{1}{2}\phi^A F_{AB}\phi^B \quad (3)$$

while the "interaction" term is supposed to be analytic in the neighbourhood of $\phi^A = 0$. The existence of a local solution of this type is guaranteed by well known theorems (see [15]. [17]).

The quantum action is then perturbatively developed as:

$$W = F + I_{cl} + \sum_{p \geq 1} \hbar^p M_p = F + I \quad (4)$$

where $I(\phi, \phi_a^*, \bar{\phi}, \pi_a)$ is the quantum generalization of the interaction part, which will include cut-off dependent terms.

We define the smoothing operator ϵ and the cut-off parameter Λ^2 , generating a second order derivative regulator of the type [7]:

$$R_B^A = (T^{-1})^{AC} F_{CB} \quad (5)$$

where the symmetric operator $(T^{-1})^{AB}$ does not depend on the fields, while:

$$\epsilon_B^A = \exp\left(\frac{R_B^A}{2\Lambda^2}\right) \quad (6)$$

The phase space is temporarily enlarged by adding the so-called "shadow" fields and antifields $\{\varphi, \varphi_a^*, \overline{\varphi}, \phi_{\varphi;a}\}$ having the same statistics as the original fields the antibracket structures being extended as well.

The "shadow" propagator O corresponding to these fields is, in perfect agreement with [9], the auxiliary quantum action being then given by:

$$\overline{W}(\phi, \phi_a^*, \overline{\phi}, \pi_a, \varphi, \varphi_a^*, \overline{\varphi}, \pi_{\varphi a}) = F((\epsilon^{-1})\phi) - \frac{1}{2}\varphi^A O_{AB}^{-1} \varphi^B + I(\phi, \phi_a^*, \overline{\phi}, \pi_a, \varphi, \varphi_a^*, \overline{\varphi}, \pi_{\varphi a}) \quad (7)$$

One can easily check that the BRST-anti-BRST symmetry that leaves the classical action \overline{S} invariant is defined by:

$$\overline{\delta}^a \phi^A = (\epsilon^2)_B^A R^{aA}(\phi + \varphi) \quad (8)$$

$$\overline{\delta}^a \varphi^A = (1 - \epsilon^2)_B^A R^{aA}(\phi + \varphi) \quad (9)$$

if the original action is invariant under the transformations:

$$\delta \phi^A = R^A(\phi) \quad (10)$$

The standard expansion of the type:

$$S = S_0 + \lambda_{Aa}^* R^{Aa}(\lambda) + \overline{\lambda}^A Y_A(\lambda) + \pi_{\lambda Aa} X^{Aa} + \dots \quad (11)$$

and the relations imposed by the classical master equations on the "structure functions" [6] allow us to identify new "coordinates", obtained from the old ones by linear transformations:

$$\lambda^A = \phi^A + \varphi^A \quad (12)$$

$$\lambda_{Aa}^* = \left[\phi_{Ba}^* (\epsilon^2)_A^B + \varphi_{Ba}^* (1 - \epsilon^2)_A^B \right] \quad (13)$$

$$\overline{\lambda}^A = \overline{\phi}^A + \overline{\varphi}^A \quad (14)$$

$$\pi_{\lambda;Aa}^* = \left[\pi_{Ba}^* (\epsilon^2)_A^B + \pi_{\varphi;Ba}^* (1 - \epsilon^2)_A^B \right] \quad (15)$$

Consequently, the action \overline{W} can be written as the sum of two terms: $W(\lambda, \lambda_a^*, \overline{\lambda}, \pi_{\lambda;a})$ and a second one, which depends on additional coordinates $\phi^A - (\epsilon^2)_B^A (\phi^B + \varphi^B)$, quadratically.

The new action is thus obtained by replacing the original fields $\phi^A, \bar{\phi}$ with their smeared versions $(\epsilon^{-1})_B^A \phi^B, (\epsilon^{-1})_B^A \bar{\phi}^B$ and by adding the shadow fields contribution defined by the propagator O^{AB} , to the action. The antifields ϕ_{Aa}^*, π_{Aa} have to be replaced in the interaction functional by λ_{Aa}^* and $\pi_{\lambda;Aa}$, respectively.

The Sp(2) generalization of the results [7] - [9] will therefore guarantee that the process does not lead to distortions of the extended BRST-anti-BRST structure. The new perturbation theory is equivalent to the initial one if the external lines φ are excluded. The aim is thus to eliminate the closed loops generated by shadow lines. We will accomplish this by using the canonical transformations derived in [17] which will set $\varphi^*, \bar{\varphi}, \pi_\varphi$ to zero and keep only the fields φ equal to their on-shell values. The solution $\varphi_q(\phi, \phi^*, \bar{\phi}, \pi)$ is then replaced in the auxiliary action $\bar{W}(\phi, \phi^*, \bar{\phi}, \pi, \varphi, \varphi^*, \bar{\varphi}, \pi_\varphi)$.

The final form of the non-local quantum action is thus the one that has to be used in the regularized perturbative calculations:

$$W_\Lambda(\phi, \phi^*, \bar{\phi}, \pi) \equiv \bar{W}(\phi, \phi^*, \bar{\phi}, \pi, \varphi_q(\phi, \phi^*, \bar{\phi}, \pi), \varphi^* = 0, \bar{\varphi} = 0, \pi_\varphi = 0) \quad (16)$$

while the expansion:

$$W_\Lambda = S_\Lambda + \sum_{p \geq 1} \hbar^p M_{p,\Lambda} \quad (17)$$

gives both the classical action and the counter-terms.

At quantum level, the extended BRST structure and its possible violations [13] are described by the regularized version of the Ward identities:

$$\frac{1}{2}(\Gamma_\Lambda, \Gamma_\Lambda)^a + V^a \Gamma_\Lambda = -i\hbar(\mathcal{A}_\Lambda^a \cdot \Gamma_\Lambda) \quad (18)$$

where Γ_Λ is the effective action associated to the regularized quantum action W_Λ .

The anomalies are still of the form:

$$\begin{aligned} \mathcal{A}_\Lambda^a(\phi, \phi^*, \bar{\phi}, \pi) &= \left[\bar{\Delta}^a W_\Lambda + \frac{i}{2\hbar} (W_\Lambda, W_\Lambda)^a \right] (\phi, \phi^*, \bar{\phi}, \pi) = \\ &= \mathcal{A}_\Lambda^a(\phi + \varphi_q, \phi^* \epsilon^2, \bar{\phi}, \pi \epsilon^2) \end{aligned} \quad (19)$$

but this time the action of the $\bar{\Delta}^a$ operators is well - defined due to the nonlocality of W_Λ .

However, both W_Λ and $\overline{\Delta}^a W_\Lambda$ will contain divergent terms. For example, in \hbar - order, one can show that $\overline{\Delta}^a S_\Lambda$ will diverge as Λ^1 .

Therefore, the regularization can not be removed at this stage, and the limit $\lim_{\Lambda \rightarrow \infty} \mathcal{A}_\Lambda^a$ is meaningful only after the divergences are subtracted and the trivial anomalies are identified.

In what follows, we will not apply a procedure similar to the one which was previously used in the field-antifield nonlocal regularization [14].

By contrast, the renormalization technique will be manifest in our approach and is based on the generating functional for 1PI vertices associated to the solution of the quantum master equations.

The “effective action” Γ and the complex terms $(\mathcal{A}_\Lambda^a \cdot \Gamma)$ which incorporate the effects of: local contributions to the anomaly, the quantum dressings of the non-trivial anomalies in the previous stages and the breakings of the master equation due to the regularization non-invariance, are treated, at each perturbative order, as the ones generated in the previous step, after the divergences subtraction.

We will therefore define:

$$\Gamma_\Lambda = \sum_{p=0} h^p \Gamma_{\Lambda R_{p-1}}^{(p)} \quad (20)$$

where each term in the expansion may be explicitly given as:

$$\Gamma_{\Lambda R_{p-1}}^{(p)} = \sum_{n=n_p} \Lambda^{-n} \Gamma_{\Lambda R_{p-1}}^{(p)n} \quad (21)$$

after $(p-1)$ steps ($p \geq 1$) have been completed by eliminating the divergences. The value of the lower limit n_p of the power series in Λ^{-1} is determined this way. We denoted by: $\Gamma_{\Lambda R_0}^{(1)} \equiv \Gamma^{(1)}$; $\Gamma^{(0)n} \equiv S_n$; $n_1 = -1$.

Even the set of equations obtained for higher orders has the formal aspect of the one given in [12], it encodes all the contributions previously described and involves well-defined expressions:

$$-i \left(\overline{\mathcal{A}}^a \circ \Gamma_{\Lambda R_{p-1}}^{(p)} \right) = \left(\Gamma_{\Lambda R_{p-1}}^{(p)}, \Gamma_{\Lambda R_{p-1}}^{(0)} \right)^a + \tilde{V}^a \Gamma_{\Lambda R_{p-1}}^{(p)} + \sum_{q=1}^{p-1} \left(\Gamma_{\Lambda R_{p-1}}^{(p)}, \Gamma_{\Lambda R_{p-1}}^{(p-q)} \right)^a \quad (22)$$

On the other hand, the inhomogeneous "consistency conditions" :

$$\left(\left(\overline{\mathcal{A}}^{\{a\}} \circ \Gamma_{\Lambda R_{p-1}}^{(p)} \right), \Gamma_{\Lambda R_{p-1}}^{(0)} \right)^{b\}} + \tilde{V}^{\{a} \left(\overline{\mathcal{A}}^{b\}} \circ \Gamma_{\Lambda R_{p-1}}^{(p)} \right) = - \sum_{q=1}^{p-1} \left(\overline{\mathcal{A}}^{\{a\}} \circ \Gamma_{R_{p-1}}^{(p)}, \Gamma_{R_{p-1}}^{(p-q)} \right)^{b\}} \quad (23)$$

($p \geq 2$) plays an important role in the divergence subtraction procedure and identification of the genuine anomalies.

Both equations (22),(23) have to be written in Λ^{-n} , for $n = -n_p, \dots, -1, 0, 1, \dots$ at each value of p , while the limit $\Lambda^{-1} \rightarrow 0$ may be taken (removing the regularization) only when this process does not generate any divergences, i.e. when the terms with poles in Λ^{-1} have been subtracted.

The starting order of the $\left(\overline{\mathcal{A}}^a \circ \Gamma_{\Lambda R_{p-1}}^{(p)} \right)$ - terms in Λ^{-1} has to be chosen such that ($p = 0$):

$$\left(\Gamma^{(0)}, \Gamma^{(0)} \right)^a + \tilde{V}^a \Gamma^{(0)} \equiv \Lambda^{-1} \alpha^{(0)a} \quad (24)$$

such that, if the regularization is removed in (24) , the master equation at classical level is recovered.

By denoting $(1/\Lambda)\alpha^{(1)a}$ the $-i \left(\overline{\mathcal{A}}^a \circ \Gamma_0^{(1)} \right)$ term, the equation (22) gives:

$$\alpha^{(1)-1a} = \left(\Gamma^{(0)0}, \Gamma^{(1)0} \right)^a + \tilde{V}^a \Gamma^{(1)0} + \left(\Gamma^{(1)-1}, \Gamma^{(0)1} \right)^a \quad (25)$$

while the equation (23) becomes:

$$\left(\alpha^{(1)-1\{a} - \left(\Gamma^{(1)-1}, \Gamma^{(0)1} \right)^{\{a}, \Gamma^{(0)0} \right)^{b\}} + \tilde{V}^{\{a} \left(\alpha^{(1)-1\{a} - \left(\Gamma^{(1)-1}, \Gamma^{(0)1} \right)^{\{a} \right)^{b\}} = 0 \quad (26)$$

and the local contribution to the one loop anomaly is identified:

$$A_1^a \equiv \alpha^{(1)-1a} - \left(\Gamma^{(1)-1}, \Gamma^{(0)1} \right)^a \quad (27)$$

In order to compute higher orders in \hbar the divergence $\Gamma^{(1)-1}$ and the trivial anomaly must be eliminated.

This aim is accomplished by writing the equation (22) at order Λ^1 :

$$0 = \left(\Gamma^{(0)0}, \Gamma^{(1)-1} \right)^a + \tilde{V}^{\{a} \Gamma^{(1)-1} \quad (28)$$

and (26) generates:

$$0 = \left(\Gamma^{(0)0}, A_1^{\{a} \right)^{b\}} + \tilde{V}^{\{a} A_1^{a\}} \quad (29)$$

The divergence $\Gamma^{(1)-1}$ and the trivial anomalies may be eliminated by an appropriate $\hbar\Lambda$ -dependent BRST- anti- BRST change of variables as the canonical ones given in [17], which will leave a total change in the effective action equal to $-\hbar\Lambda\Gamma^{(1)-1}$ if $\mu_a = \frac{i\Lambda}{2}\epsilon_{ab}s^b\gamma$, $\tilde{\mu}_a = i\Lambda s^a\alpha_1$ where γ, α_1 are given by $\Gamma^{(1)-1} = \frac{1}{2}\epsilon_{ab}s^as^b\gamma + \gamma_0$ and $A_1^a = s^a\alpha_1 + \alpha_{01}^a$ but with \hbar^2/Λ^{-2} contributions that will propagate to the next level.

The first result on the effective action $\Gamma_{\Lambda R_1}$ is then of the following form:

$$\Gamma_{\Lambda R_1} = \Gamma_{\Lambda R_1}^0 + \hbar \sum_{n=0} \Lambda^{-n} \Gamma_{\Lambda R_1}^{(1)n} + \hbar^2 \sum_{n=-2} \Lambda^{-n} \Gamma_{\Lambda R_1}^{(2)n} + O(\hbar^3) \quad (30)$$

while the new $-i(\overline{\mathcal{A}}^a \circ \Gamma_{\Lambda R_1}^{(2)})$ contain two type of terms: $\Lambda^{-1}\alpha_{R_1}^{(2)a}$ and the renormalization dressing of the non-trivial one loop anomaly $\Lambda^{-1}\overline{\alpha}_{R_1}^{(1)a}$. It corresponds, at $\Lambda \rightarrow \infty$, to the genuine anomaly $\mathcal{A}_1^{(a)}$ which satisfies the homogeneous consistency condition:

$$(S, \mathcal{A}_1^{\{a\}})^{b\}} = 0 \quad (31)$$

In the next step of our algorithm, the equation (22) is reproduced for $\hbar^2, \Lambda^0, \Lambda^1, \Lambda^2$ and we can identify the following local contribution to the anomaly:

$$\begin{aligned} \alpha_{R_1}^{(2)-1a} + \overline{\alpha}_{R_1}^{(1)-1a} &= \frac{1}{2} \left(\Gamma_{R_1}^{(1)0}, \Gamma_{R_1}^{(1)0} \right)^a + \tilde{V}^a \Gamma_{R_1}^{(2)0} \\ &+ \left(\Gamma_{R_1}^{(0)0}, \Gamma_{R_1}^{(2)0} \right)^a + \left(\Gamma_{R_1}^{(0)1}, \Gamma_{R_1}^{(2)-1} \right)^a + \left(\Gamma_{R_1}^{(0)2}, \Gamma_{R_1}^{(2)-2} \right)^a \end{aligned} \quad (32)$$

with:

$$A_2^a \equiv \alpha_{R_1}^{(2)-1a} - \left(\Gamma_{\Lambda R_1}^{(0)1}, \Gamma_{\Lambda R_1}^{(2)-1} \right)^a + \left(\Gamma_{\Lambda R_1}^{(0)2}, \Gamma_{\Lambda R_1}^{(2)-2} \right)^a \quad (33)$$

as well as the "triviality" conditions:

$$0 = \left(\Gamma^{(0)0}, A_2^{\{a\}} \right)^{b\}} + \tilde{V}^{\{b} A_2^{a\}} \quad (34)$$

and:

$$0 = \left(\Gamma^{(0)0}, \Gamma_{\Lambda R_1}^{(2)-2} \right)^a + \tilde{V}^a \Gamma_{\Lambda R_1}^{(2)-2} \quad (35)$$

while the Λ^1 divergence satisfies:

$$\left(\Gamma^{(0)0}, \Gamma_{\Lambda R_1}^{(2)-1} \right)^a + \tilde{V}^a \Gamma_{\Lambda R_1}^{(2)-1} = \alpha_{R_1}^{(2)-2a} - \left(\Gamma^{(0)1}, \Gamma_{\Lambda R_1}^{(2)-2} \right)^a \quad (36)$$

Once again, the divergences must be eliminated, one by one, by the same kind of BRST-anti-BRST transformations on S , leading us to a finite $\Gamma_{\Lambda R_2}$ and to a modified $\Lambda^{-1}\alpha_{R_2}^{(2)a}$. The procedure may be further applied to obtain a completely regularized and subtracted effective action $\Gamma_{\Lambda R_\infty}$ which respects the anomalous equations.

An instructive example of the way our method works is to calculate the BRST and anti-BRST anomalies, in the first order of the perturbation theory, for the theory of W_2 gravity. Our procedure then will provide, starting with the classical action:

$$S_0 = \frac{1}{2\pi} \int d^2x \left(\partial\phi\bar{\partial}\phi - h(\partial\phi)^2 \right) \quad (37)$$

the anomalous contribution given by:

$$\mathcal{A}_a^1 = \int d^2x c^a \partial^3 h \quad (38)$$

up to a numerical factor, and which respect the condition (31) and are in full agreement with the result of [18].

The non-local regularization and renormalization of the $\text{Sp}(2)$ symmetric BV formalism is thus able to determine the non-trivial higher order anomalies, in any order of the perturbation theory.

3 Conclusions

A systematic treatment was proposed in this paper for completing the regularization and renormalization stages of the $\text{Sp}(2)$ symmetric quantisation scheme. The extension of the non-local regularization technique proved to be effective in solving the higher loop anomaly problems for both BRST and anti BRST sectors.

The main role in this procedure is played by the perfectly symmetrical structure of the extended phase space [11]. However, the existence of Darboux like coordinates is not enforced by a general theorem in the infinite dimensional case, such that the method is restricted to the problems which admit such a construction. The solution is provided by the complex "triplectic quantisation" [10] which in turn extends the phase space and the hierarchy of theory levels.

On the other hand, the double-cohomology analysis of the conditions imposed to the classical action and to the counter-terms were not extensively studied yet and one should expect them to provide major clues in the calculation of higher loop anomalies as well.

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